Soluții

① a) $2\cdot 2-2+1=m \Rightarrow m=3$

$n\cdot 2+2-2\cdot 1=4 \Rightarrow n=2$

b) $\det(A)=\begin{vmatrix}1 & 2 & -3\\ 2 & -1 & 1\\ n & 1 & -2\end{vmatrix}=2-6+2n-3n-1+8=5-n$

Sistemul are sol. unică $\Leftrightarrow \det(A)\neq 0 \Leftrightarrow n\neq 5$

c) Sistem compatibil nedeterminat $\Rightarrow \det A=0$ și $\operatorname{rang} A=\operatorname{rang}\bar{A} \Rightarrow n=5$.

Cum $\operatorname{rang} A=2$ $\left(d_1=\begin{vmatrix}1 & 2\\ 2 & -1\end{vmatrix}=-5\neq 0\right) \Rightarrow \operatorname{rang}\bar{A}=2$

$\Rightarrow \begin{vmatrix}1 & 2 & 3\\ 2 & -1 & m\\ n & 1 & 4\end{vmatrix}=0 \Rightarrow \begin{vmatrix}1 & 2 & 3\\ 2 & -1 & m\\ 5 & 1 & 4\end{vmatrix}=0 \Rightarrow 2+10m-1-m=0$

$\Rightarrow m=-\frac{1}{9}$.

② a) $f'(x)=\frac{x^2+1-2x^2}{(x^2+1)^2}=\frac{1-x^2}{(x^2+1)^2}$

b) $\lim\limits_{x\to\infty}\left[\left(1+\frac{x}{x^2+1}\right)^{\frac{x^2+1}{x}}\right]^{\frac{x}{x^2+1}\cdot x}=e^{\lim\limits_{x\to\infty}\frac{x^2}{x^2+1}}=e^1=e$

c) $f'(x)=0 \Rightarrow x=\pm 1 \Rightarrow$

x	$-\infty$		-1		1		∞
$f'(x)$		$-$	0	$+$	0	$-$	
$f(x)$	0	$\searrow$	$-\frac{1}{2}$	$\nearrow$	$\frac{1}{2}$	$\searrow$	0

$\Rightarrow \operatorname{Im}(f)=\left[-\frac{1}{2},\frac{1}{2}\right]$

③ a) $f(0-0)=\lim\limits_{\substack{x\to 0\\ x<0}}2^x=1$; $f(0+0)=\lim\limits_{\substack{x\to 0\\ x>0}}\sqrt{|x-1|}=1$

$f(0)=1 \Rightarrow f$ continuă în 0

f cont pe $(-\infty,0)$, $(0,\infty)$ ca f. elem.

$\Rightarrow f$ cont. pe $\mathbb{R}$.

b) $f'(x)=\begin{cases}2^x\ln 2, & x<0\\ \frac{-1}{2\sqrt{1-x}}, & x\in(0,1)\\ \frac{1}{2\sqrt{x-1}}, & x\in(1,\infty)\end{cases}$

$f'_s(0)=\ln 2$, $f'_d(0)=-\frac{1}{2}$, f cont în 0 $\Rightarrow x_0=0$ punct unghiular

$f'_s(1)=-\infty$, $f'_d(1)=+\infty$, f cont în 1 $\Rightarrow x_0=1$ punct de întoarcere

c) $t: y-f(5)=f'(5)(x-5) \Rightarrow y-2=\frac{1}{4}(x-5)$.

④ a) $f'(x)=\frac{\frac{1}{x}\cdot\sqrt{x}-\ln x\cdot\frac{1}{2\sqrt{x}}}{x}=\frac{2-\ln x}{2x\sqrt{x}}$

b) $f'(x)=0 \Rightarrow x=e^2 \Rightarrow$

x	0		e^2		∞
$f'(x)$		$+$	0	$-$	
$f(x)$	$-\infty$	$\nearrow$	$\frac{2}{e}$	$\searrow$	0

c) $3<5<e^2$

f cresc pe $(0,e^2)$ $\Rightarrow f(3)<f(5) \Rightarrow \frac{\ln 3}{\sqrt{3}}<\frac{\ln 5}{\sqrt{5}}$

$\Rightarrow \sqrt{5}\ln 3<\sqrt{3}\ln 5 \Rightarrow 3^{\sqrt{5}}<5^{\sqrt{3}}$.