

Soluții

1. a) $5 + 0 + \frac{\log_3 81}{\log_3 27} - 3 = 5 + \frac{4}{3} - 3 = \frac{10}{3}$

b) $\sqrt{16+6\sqrt{7}} = \sqrt{9+6\sqrt{7}+7} = \sqrt{(3+\sqrt{7})^2} = 3+\sqrt{7}$

$\Rightarrow \sqrt{16+6\sqrt{7}} + \sqrt{16-6\sqrt{7}} = 3+\sqrt{7} + 3-\sqrt{7} = 6 \in \mathbb{Q}$

c) $\log_{\frac{45}{60}} 60 = \frac{\log_2 60}{\log_2 45} = \frac{\log_2 (2^2 \cdot 3 \cdot 5)}{\log_2 (3^2 \cdot 5)} = \frac{2 + \log_2 3 + \log_2 5}{2 \log_2 3 + \log_2 5} = \frac{2 + \frac{1}{6} + a}{\frac{2}{6} + a}$

2. a) $(2^x)^2 - 9 \cdot 2^x + 8 = 0$. Not. $2^x = y, y > 0 \Rightarrow y^2 - 9y + 8 = 0$
 $\Rightarrow y_1 = 1, y_2 = 8 \Rightarrow 2^x = 1$ sau $2^x = 8 \Rightarrow x_1 = 0$ sau $x_2 = 3$

b) CE: $2x-1 > 0, x+22 > 0 \Rightarrow x \in (\frac{1}{2}, \infty)$.

Ec. devine: $\log_3 \frac{2x-1}{x+22} = -1 \Rightarrow \frac{2x-1}{x+22} = 3^{-1} = \frac{1}{3} \Rightarrow$

$\Rightarrow 6x-3 = x+22 \Rightarrow 5x = 25 \Rightarrow x = 5 \in (\frac{1}{2}, \infty)$

c) $(7^x)^2 + 7^x \cdot 7 + 3m - 2 = 0$. Not. $7^x = y, y > 0 \Rightarrow$

\Rightarrow ec. devine: $y^2 + 7y + 3m - 2 = 0$.

$\bar{I} \Delta < 0 \Leftrightarrow 49 - 12m + 8 < 0 \Leftrightarrow 57 < 12m \Leftrightarrow m > \frac{19}{4}$

$\bar{II} \Delta = 0 \Rightarrow m = \frac{19}{4} \Rightarrow y_1 = y_2 = -\frac{b}{2a} = -\frac{7}{2} < 0 \Rightarrow$ ec. nu are sol. $\Rightarrow m = \frac{19}{4}$

$\bar{III} \Delta > 0 \Leftrightarrow m < \frac{19}{4}$. Ptr. ca ecuatia in x sa nu aiba solutii trebuie ca $y_1, y_2 \leq 0 \Leftrightarrow$

$\Leftrightarrow \begin{cases} -7 \leq 0 \\ 3m - 2 \geq 0 \end{cases} \Leftrightarrow m \geq \frac{2}{3}$ si cum $m < \frac{19}{4} \Rightarrow m \in [\frac{2}{3}, \frac{19}{4})$

Dei $\bar{I}, \bar{II}, \bar{III} \xrightarrow{\text{"U"}} m \in [\frac{2}{3}, \infty)$.

3. a) Fie $y \in \mathbb{R}$. Cautam $x \in \mathbb{R}$ a.p. $f(x) = y \Leftrightarrow$

$\Leftrightarrow 3x - 5 = y \Leftrightarrow x = \frac{y+5}{3}$ sol. unica $\Rightarrow f$ bijectivă

$\Rightarrow f$ inversabila $f: \mathbb{R} \rightarrow \mathbb{R}, f^{-1}(x) = \frac{x+5}{3}$.

Ec. $f(x) + f^{-1}(x) = 7 \Leftrightarrow 3x - 5 + \frac{x+5}{3} = 7 \Leftrightarrow$

$\Leftrightarrow 9x - 15 + x + 5 = 21 \Leftrightarrow 10x = 31 \Leftrightarrow x = 3,1$

b) $-\frac{b}{2a} = \frac{1}{2} \Rightarrow$

x	$[0$	$\frac{1}{2}$	$4)$
$f(x)$	$[1$	$\frac{3}{4}$	$13)$

$\Rightarrow \text{Im}(f) = [\frac{3}{4}, 13) \Rightarrow f$ surjectivă $\Leftrightarrow M = [\frac{3}{4}, 13)$