Soluții

1. a) $S = 2 + \frac{2}{3} + \sqrt{5} - 2 = \frac{2}{3} + \sqrt{5}$

b) $|2x-3| \le 5 \Leftrightarrow -5 \le 2x-3 \le 5 \ |+3 \Leftrightarrow -2 \le 2x \le 8$

$\Leftrightarrow x \in [-1;4]$

c) $\frac{a^2}{5} + \frac{b^2}{7} \ge \frac{(a+b)^2}{12} \Leftrightarrow 84a^2 + 60b^2 \ge 35a^2 + 70ab + 35b^2$

$\Leftrightarrow 49a^2 - 70ab + 25b^2 \ge 0 \Leftrightarrow (7a-5b)^2 \ge 0$ (A)

2. a) $x \ge 0 \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow x_1 = 1,\ x_2 = 2$

$x < 0 \Rightarrow x^2 + 3x + 2 = 0 \Rightarrow x_1 = -1,\ x_2 = -2.$

b) $\Delta \ge 0 \Leftrightarrow (2m+1)^2 - 4(m^2 - 5m + 9) \ge 0 \Leftrightarrow 24m - 35 \ge 0$

$\Leftrightarrow m \in \left[\frac{35}{24}, \infty\right)$

c) $S = y_1 + y_2 = x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = 3^2 - 2\cdot(-7) = 23$

$P = y_1 \cdot y_2 = x_1^2 \cdot x_2^2 = (x_1x_2)^2 = (-7)^2 = 49$

$\Rightarrow$ ec. este $y^2 - Sy + P = 0 \Rightarrow y^2 - 23y + 49 = 0$

3. a) $p \Leftrightarrow 2 + 2\sqrt{22} + 11 \ge 3 + 2\sqrt{30} + 10 \Leftrightarrow \sqrt{22} \ge \sqrt{30}$ (F)

b) I $p(1)$: $\frac{1}{1\cdot 3} = \frac{1}{2\cdot 1+1} \Leftrightarrow \frac{1}{3} = \frac{1}{3}$ A

II pp. $p(k)$ adev. și dem. $p(k+1)$: $\frac{1}{1\cdot 3} + \dots + \frac{1}{(2k+1)(2k+3)} =$

$= \frac{k+1}{2k+3} \overset{p(k)A,\ 2k+3}{\Leftrightarrow} \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)} \overset{2k+1}{=} \frac{k+1}{2k+3} \Leftrightarrow k(2k+3) + 1 = (k+1)(2k+1)$

$\Leftrightarrow 2k^2 + 3k + 1 = 2k^2 + 3k + 1$ (A). Din I și II $\Rightarrow (\forall n)\ p(n)$ adev.

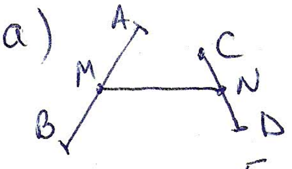
c) Pp. că $\exists\ x, y \in \mathbb{Z}$ a.î. $x^2 - y^2 = 6 \Rightarrow (x-y)(x+y) = 6$

$\Rightarrow \begin{cases} x-y=-6 \\ x+y=-1 \end{cases}$ sau $\begin{cases} x-y=-3 \\ x+y=-2 \end{cases}$ sau $\begin{cases} x-y=-2 \\ x+y=-3 \end{cases}$ sau $\begin{cases} x-y=-1 \\ x+y=-6 \end{cases}$

sau $\begin{cases} x-y=1 \\ x+y=6 \end{cases}$ sau $\begin{cases} x-y=2 \\ x+y=3 \end{cases}$ sau $\begin{cases} x-y=3 \\ x+y=2 \end{cases}$ sau $\begin{cases} x-y=6 \\ x+y=1 \end{cases}$

În toate cazurile obținem $x \notin \mathbb{Z}$. Contradicție!

4. a)

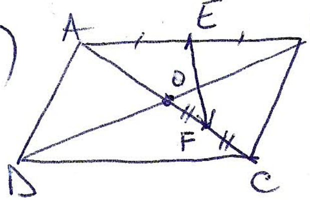


$2\vec{MN} = \vec{MN} + \vec{MN} = \vec{MA} + \vec{AC} + \vec{CN} + \vec{MB} + \vec{BD} + \vec{DN} =$

$= \vec{AC} + \vec{BD} + (\vec{MA} + \vec{MB}) + (\vec{CN} + \vec{DN}) =$

$= \vec{AC} + \vec{BD} + \vec{0} + \vec{0} = \vec{AC} + \vec{BD}.$

b)



$\vec{AC} = \vec{AB} + \vec{AD}$

$\vec{AO} = \frac{1}{2}\vec{AC} = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AD}$

$\vec{EF} = \vec{EA} + \vec{AF} = \frac{1}{2}\vec{BA} + \frac{3}{4}\vec{AC} = -\frac{1}{2}\vec{AB} +$

$+ \frac{3}{4}(\vec{AB} + \vec{AD}) = \frac{1}{4}\vec{AB} + \frac{3}{4}\vec{AD}$

c) $\vec{G_1G_2} = \vec{G_1O} + \vec{OG_2} = \vec{OG_2} - \vec{OG_1} = \frac{\vec{OA_2} + \vec{OB_2} + \vec{OC_2}}{3} -$

$- \frac{\vec{OA_1} + \vec{OB_1} + \vec{OC_1}}{3} = \frac{\vec{A_1A_2} + \vec{B_1B_2} + \vec{C_1C_2}}{3}$